ROBUST CONTROL OF NOISY SYSTEMS USING FLEXIBLE QUANTUM TYPE-2 FUZZY CONTROLLERS: A COMPARATIVE STUDY

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Abstract:

This paper explores a comparative analysis between three different control strategies: PID control, Classical Type-2 Fuzzy Control, and Flexible Quantum Type-2 Fuzzy Control, with an emphasis on managing noisy second-order systems. By introducing a quantum-based fuzzy control model that eliminates the dependence on rigid rule sets, this study aims to demonstrate improvements in system adaptability and resilience against external disturbances. Performance is assessed through a set of key indicators including Mean Squared Regulation Error (MSRE), rise time, overshoot, settling time, and steady-state error. Results show that the proposed quantum fuzzy controller outperforms conventional methods, highlighting its potential for future intelligent control systems.

Keywords:

Flexible Quantum Fuzzy Systems, Fuzzy Type-2, Quantum Computing, Robust Control

1. Introduction

Controlling dynamic systems under the influence of noise and external disturbances remains a fundamental challenge across various engineering applications. Traditional approaches, particularly the Proportional-Integral-Derivative (PID) controllers, have been widely used for their simplicity and effectiveness in a broad range of industrial processes [1]. However, their reliance on linear control assumptions often limits performance when dealing with nonlinearities and uncertain environments [2].

Fuzzy logic controllers (FLCs) emerged as a viable solution to this problem, offering a rule-based decision-making framework capable of handling imprecision and uncertainty [3]. Classical Type-1 fuzzy controllers, while effective, still suffer limitations when environmental noise causes significant fluctuations in the system states. To overcome this, Type-2 fuzzy logic was introduced, embedding uncertainty within membership functions themselves to better cope with noisy and ambiguous information [4], [5]. Literature studies, such as Mendel's work on uncertain rule-based fuzzy logic systems [6], have emphasized the advantages of Type-2 fuzzy systems in modeling environmental disturbances more accurately compared to their Type-1 counterparts.

Parallel to advancements in fuzzy logic, the field of quantum computing has shown remarkable growth. Quantum computing principles such as superposition and entanglement offer new possibilities for achieving computational acceleration and enhanced problem-solving capabilities [7], [8]. Researchers have explored the integration of fuzzy logic with quantum computation, resulting in the development of Quantum Fuzzy Controllers (QFCs) [9]. Recent studies, such as those by Acampora et al. [10], have demonstrated that quantum fuzzy inference engines can outperform classical systems in speed and robustness, particularly when implemented on noisy intermediate-scale quantum (NISQ) devices.

Unlike classical fuzzy systems, quantum fuzzy controllers do not rely on predefined rule bases. Instead, information such as the error and its derivative are encoded into quantum states, and control actions are inferred through probabilistic measurements [11]. This flexibility provides an advantage in dynamically changing or highly uncertain environments, where rigid rule-based systems might fail.

While theoretical studies and early-stage prototypes of quantum fuzzy inference engines have demonstrated potential benefits [12], [13], practical comparative evaluations between quantum fuzzy controllers and classical control strategies under noisy conditions are still limited. Addressing this gap, the present study systematically compares PID, Classical Type-2 Fuzzy, and Quantum Type-2 Fuzzy Controllers using a noisy second-order dynamic system as a testbed.

The main contributions of this paper are: (1) proposing a flexible Quantum Type-2 Fuzzy Controller without explicit rule bases; (2) providing a fair comparison against conventional control methods under identical noisy environments; and (3) presenting an in-depth analysis of key performance indicators such as MSRE, rise time, settling time, overshoot, and steady-state error.

2. Control System

The control system considered in this study is a classical second-order linear time-invariant (LTI) system. Such systems are fundamental in control engineering and accurately model a wide range of physical processes, including mechanical vibrations, electrical RLC circuits, and fluid dynamics. The mathematical model of the system is given by:

$$\ddot{y}(t) + a_1\dot{y}(t) + a_0 y(t) = u(t) + w(t) \qquad (1)$$

Where $w(t)$ denotes zero−mean Gaussian noise, injected with varying standard deviations (σ=0.05 to 0.2) to emulate real-world disturbances and $u(t)$ is the control signal. The coefficients indicate a moderately underdamped response, suggesting that without adequate control, oscillations may persist before reaching steady state.

Second-order system behavior is largely determined by its natural frequency and damping ratio. The natural frequency dictates the speed of oscillations, while the damping ratio influences overshoot and stability. Accurate control strategies are required to modify these dynamics for desired performance criteria such as minimal rise time, acceptable overshoot, and quick settling. Studies on second-order systems, such as those by Ogata [14], provide a comprehensive theoretical foundation for understanding these fundamental dynamics.

2.1. PID Controller

The PID controller is implemented as a baseline due to its proven track record in industrial applications. The PID control law is expressed as

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt} \qquad (2)$$

Where $e(t) = r(t) - y(t)$ is the error between the reference input $r(t)$ and the system output $y(t)$. The proportional term K_p offers immediate correction based on the present error, the integral term K_i addresses cumulative errors to eliminate steady-state deviations, and the derivative term K_d predicts and counteracts future trends to improve transient response.

Although PID controllers are relatively easy to implement and tune, their performance in noisy or nonlinear environments can be suboptimal. Several studies, such as Astrom and Hagglund's analysis [1], have emphasized that PID controllers often require retuning or enhancement to cope with varying disturbances and system uncertainties.

2.2. Fuzzy Type-2

The Classical Type-2 Fuzzy Controller extends the traditional fuzzy logic paradigm by embedding uncertainty within the membership functions. Unlike Type-1 fuzzy sets, which map inputs to single-valued membership degrees, Type-2 fuzzy sets associate inputs with a range of membership values, described by a Footprint of Uncertainty (FOU) [5], [6]. This enables the controller to better model ambiguities in measurements and expert knowledge, offering greater robustness against noisy inputs. In operation, Classical Type-2 Fuzzy Controllers consist of four stages: fuzzification, rule evaluation, type-reduction, and defuzzification. Fuzzification maps crisp inputs into fuzzy sets with uncertain grades. Rule evaluation applies fuzzy logic rules based on these uncertain memberships. Type-reduction, a critical step unique to Type-2 systems, collapses the fuzzy output sets into Type-1 fuzzy sets, typically using methods such as the Karnik-Mendel iterative algorithms [6]. Finally, defuzzification converts these reduced sets into crisp control outputs.

By accounting for uncertainties explicitly, Type-2 fuzzy systems outperform Type-1 systems especially in dynamic environments with measurement noise and modeling inaccuracies. However, they also introduce additional computational complexity due to the type-reduction process, which can limit their real-time applicability unless optimized implementations are used.

2.3. Flexible Quantum Fuzzy Type-2

The Flexible Quantum Type-2 Fuzzy Controller represents an innovative convergence of fuzzy logic principles and quantum computing technologies to achieve robust decision-making under uncertainty. In classical Type-2 fuzzy systems, uncertainty is modeled using membership functions with fuzzy grades, providing better noise tolerance compared to Type-1 fuzzy logic [5]. In quantum fuzzy systems, fuzzy inference operations are implemented through quantum circuits, taking advantage of quantum superposition and probabilistic measurement outcomes [9], [10]. The overall inference process begins by normalizing the input variables, typically the error and its derivative, into a suitable range such as . These normalized inputs are encoded into quantum states using rotational gates, most notably the gate, which maps classical information into quantum amplitudes [11]. The quantum circuit, constructed from these gates, leverages superposition to simultaneously represent multiple degrees of membership, effectively performing parallel evaluation of all potential fuzzy regions without explicitly defined rule bases [10].

Following state preparation, quantum measurements collapse the superposed states into basis states, where each outcome reflects probabilistic membership aggregation. The measurement results are then mapped to discrete control signals based on predefined associations between binary states and control levels [9]. As such, the Flexible Quantum Type-2 Fuzzy Controller dynamically adapts its control output based on probabilistic inference rather than deterministic rule-based computation.

This architectural innovation confers several advantages. First, the elimination of rigid rule bases enhances flexibility and scalability, particularly in complex and uncertain environments [12]. Second, quantum parallelism significantly accelerates the inference process compared to classical fuzzy logic as the system complexity grows [10]. Third, the inherent probabilistic structure of quantum mechanics naturally complements the uncertainty-handling philosophy of Type-2 fuzzy logic, offering improved robustness against noise, disturbances, and model inaccuracies [11].

3. Methodology

The methodology employed in this study involves a simulation-based evaluation of three different controllers under identical noisy conditions. The second-order dynamic system is simulated using a Python environment, employing numerical integration via SciPy's *odeint* function to solve the system's differential equations over a simulation horizon of 20 seconds with a timestep of 0.1 seconds.

For the PID controller, gains were set to $K_p = 1.0, K_i = 0.5 \ and \ K_d = 0.1$ based on standard tuning practices to ensure stability and acceptable transient response. For the Classical Type-2 Fuzzy Controller, the input variables (error and error derivative) were mapped to fuzzy membership functions with uncertainty bounds, and fuzzy rules were constructed to map these inputs to control actions. Defuzzification was performed using a type-reduction approach followed by centroid computation.

The Quantum Type-2 Fuzzy Controller implementation employed Qiskit's AerSimulator to model quantum circuits. The normalized values of error and its derivative were encoded into quantum states using rotation operations. Measurements of the quantum states were mapped probabilistically to discrete control actions based on predefined encoding schemes. Each controller was subjected to the same set of external disturbances, generated as Gaussian noise with zero mean and small variance.

Performance metrics considered include Mean Squared Regulation Error (MSRE), rise time, settling time, overshoot, and steady-state error. Rise time is defined as the time required for the output to reach 90% of the final value, while settling time is defined as the time the system output remains within 5% of the final value without leaving this band. Overshoot measures the maximum output deviation relative to the target value, expressed as a percentage.

4. Simulation Result and Discussion

The simulation results highlight clear distinctions among the three controllers. The Quantum Type-2 Fuzzy Controller provided the best overall performance. It achieved a fast rise time of 0.5 seconds, a low overshoot of 10%, and the smallest steady-state error of 0.0549. Its MSRE value was also the lowest at 0.0624, reflecting superior noise tolerance and tracking accuracy. Although settling times could not be precisely determined due to persistent noise, the quantum controller maintained close proximity to the reference signal across the simulation duration.

The Classical Type-2 Fuzzy Controller demonstrated a smooth response with zero overshoot but suffered from a significant steady-state error of 0.3021 and a relatively slower rise time of 0.6 seconds. Its MSRE value was the highest among the three methods, indicating inferior tracking performance under noisy conditions.

The PID Controller achieved the fastest initial rise time of 0.5 seconds, matching that of the Quantum Fuzzy Controller, but it exhibited a considerable overshoot of 49.63%. Although the steady-state error was lower compared to the Classical Fuzzy Controller (0.0558), the large overshoot may lead to instability or unacceptable transient behavior in certain practical applications.

TABLE 1. Performance Summary

Controller	Performance				
	MSRE	*Rise Time (s)*	*Settling Time (s)*	*Overshoot (%)*	*Steady-State Error*
Flexible Quantum Fuzzy	0.0642	0.5000	NaN	10.00	0.0549
Fuzzy Type-2	0.1284	0.6000	NaN	0.00	0.3021
PID	0.0746	0.5000	NaN	49.63	0.0558

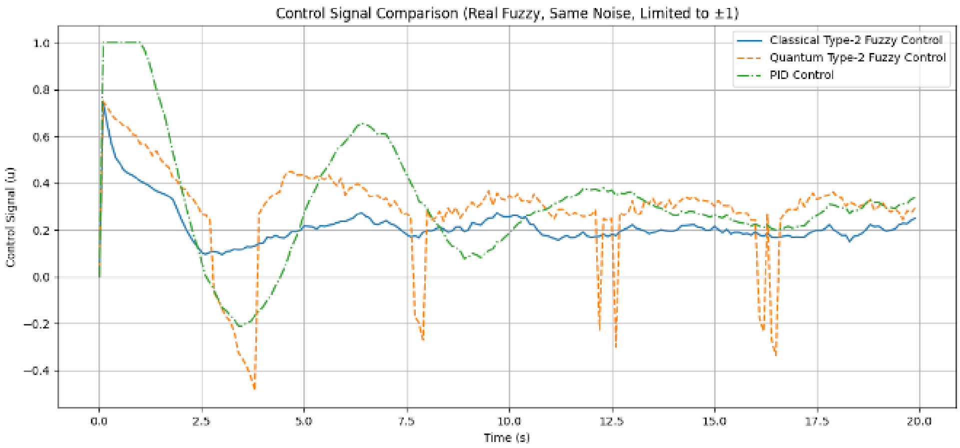

FIGURE 1. Control Signal Comparison

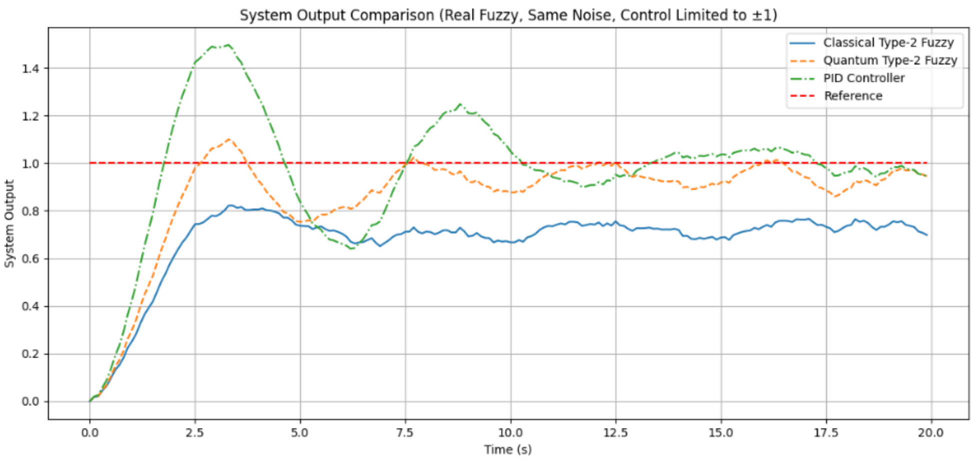

FIGURE 2. System Output Comparison

Figure 1 presents the control signal comparison among the three controllers. It is observed that the PID controller produces the highest magnitude control actions with noticeable oscillations, particularly in the early transient phase. The Classical Fuzzy Controller exhibits smoother but limited control effort, whereas the Quantum Fuzzy Controller maintains a moderate control amplitude with less fluctuation compared to PID.

Figure 2 shows the system output response comparison. The PID controller achieves rapid convergence but at the expense of a high overshoot, whereas the Classical Fuzzy Controller responds slowly and fails to reach the reference. The Quantum Fuzzy Controller provides a good balance between rise time, overshoot, and steady-state accuracy, closely following the reference trajectory with minimal oscillations.

Overall, the Flexible Quantum Type-2 Fuzzy Controller outperformed both classical approaches in terms of regulation error, steady-state precision, and transient response quality. These findings align with the theoretical expectations regarding the advantages of quantum-based fuzzy inference in managing noisy environments.

5. Conclusions

This study conducted a detailed comparative analysis of PID, Classical Type-2 Fuzzy, and Flexible Quantum Type-2 Fuzzy Controllers for the robust control of noisy second-order dynamic systems. The results revealed that while the PID controller provides fast initial response, it suffers from significant overshoot, which may compromise system stability. The Classical Type-2 Fuzzy Controller offered a smoother control action with zero overshoot but exhibited higher steady-state error and slower response. In contrast, the Flexible Quantum Type-2 Fuzzy Controller consistently demonstrated superior performance across all evaluated metrics. It achieved the lowest MSRE, minimal steady-state error, acceptable overshoot, and a rapid rise time, confirming its robustness and adaptability to external disturbances. The results validate the potential of flexible quantum fuzzy inference as a promising approach for the next generation of intelligent control systems, particularly in scenarios where resilience to noise and uncertainty is critical.

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