ROBUST CLUSTERING VIA MULTI-VIEW LOW-RANK REPRESENTATION

CHUAN-WU YANG[1,2], YUE ZHAO[⋆,3], YU-LONG WANG[4], RAN DUAN[⋆,1], XIN-GE YOU[2]

[1]Research Institute of Information Technology of National Immigration Administration, Beijing, 100176, China
[2]School of Electronic Information and Communications, Huazhong University of Science and Technology, Wuhan, 430074, China
[3]College of Police Information, Shandong Police College, Ji'nan, 250014, China
[4]College of Informatics, Huazhong Agricultural University, Wuhan, 430070, China
⋆Corresponding author. The first corresponding author is Yue Zhao.
E-MAIL: chuanwuyang@hust.edu.cn, zhaoy@sdpc.edu.cn, wangyulong6251@gmail.com,
duandr@aliyun.com, youxg@hust.edu.cn

Abstract:

Multi-view clustering aims to obtain a unified feature representation by integrating data from multiple views, enabling accurate clustering of data points in low-dimensional subspaces. However, existing approaches generally overlook the issue of data noise in some views when constructing a common feature representation by combining multi-view information, resulting in poor robustness of clustering models. To address this challenge, this paper proposes a robust multi-view low-rank representation clustering algorithm based on structural consistency. The method effectively extracts consistent information among different views by mining the similarities between samples to construct a shared structured matrix, which guides the learning of self-representation matrices for each view. Meanwhile, to mitigate the impact of noisy views on model performance, a structured weighting strategy is proposed. By calculating the differences between the self-representation matrices of each view and the shared structured matrix, view weights are determined to suppress the interference of sample noise. Experimental results demonstrate that, compared with existing methods, the proposed algorithm exhibits superior performance in multi-view clustering tasks with noisy data. Especially in complex scenarios where view data is damaged, it can significantly improve clustering accuracy and stability, providing a more reliable solution for multi-view clustering analysis.

Keywords:

Multi-view clustering; Low-rank representation; Structural consistency; Weighting strategy

1. Introduction

With the rapid development of information and communication technology, the acquisition and transmission of data is becoming more and more convenient. The scale of the data increases rapidly. Big data attracts increasing attention and becomes an important research topic in the field of artificial intelligence as it contains a large amount of useful information. Data Mining generally aims to discover patterns and knowledge hidden in a large amount of data through relevant algorithms [8]. Depending on different applications, data mining can be roughly divided into tasks such as regression analysis [9], classification [10], clustering [11], etc. The performance of different task models largely depends on the data representations (features) they utilize, and a good representation learning can more easily extract useful information from the data for classification or prediction [12]. Therefore, researchers have conducted extensive research on representation learning tasks, achieving a series of related researches and applying them to related fields such as speech recognition [13], image processing [14], etc.

In practical applications, noise of data exists in the representation learning tasks extensively, and many works have tried to mitigate the impact of noise on the learning tasks. Sparse Subspace Clustering (SSC) [15] learns a sparse self-representation matrix so that samples can be linearly represented by samples in the same subspace, thus dividing the data into different subspaces. However, Sparse Subspace Clustering is easily affected by the noise. Therefore, Low-Rank Representation (LRR) [16] is proposed to reduce the influence of noises and outliers by constraining the rank of the representation matrix, and the learned subspace is more sparse. Although LRR improves the robustness of the model by decomposing errors and constraining them with norm constraints, complex noise can still destroy the intrinsic structure of the data. Therefore, many works have tried to use the abundant information of multi-view data to mitigate the negative influence of complex noises. The corresponding multi-view subspace representation learning methods are proposed.

Multi-view data aims to describe the object from different aspects, while multi-view subspace clustering is proposed by extending the subspace clustering into multi-view scenario [17]. The purpose of multi-view subspace clustering is to learn a unified subspace representation from the multiple subspaces of multi-view data, in order to handle high-dimensional data more easily when establishing clustering models by using the multiple information of different views. The multi-view subspace clustering method assumes that multiple views are derived from the same latent space, and the key problem is to learn a consistent representation of multiple views. Most existing methods first learn a unified subspace representation and then cluster the unified subspace representation. For example, Chaudhuri et al. [18] proposed a multi-view subspace learning method based on canonical correlation analysis (CCA), which uses canonical correlation analysis to obtain a low-dimensional representation of high-dimensional data from each view and then aligns the low-dimensional representation for clustering. Guo [19] proposed a subspace representation learning method which realizes clustering on the learned common subspace representation of the multi-view data. Wang et al. [20] performed subspace clustering on a common view of multiple views. Gao et al. [21] performed subspace clustering in each view and ensured consistency of clustering between different views by using common metrics. In addition, the sparse subspace clustering-based method [22] uses sparse representation to construct the affinity matrix of each view, then fuses the affinity matrices to obtain a multi-view affinity matrix for clustering. Tensor decomposition-based methods [23] [24] [25] combined the self-representation matrices of each view to obtain a three-dimensional representation matrix, and apply tensor sparsity constraints to the representation matrix to ultimately obtain a fused self-representation matrix. Low-Rank representation-based methods [26] [27] [28] obtained complementary features of different views by constructing a low-rank representation of each view and obtained the general low-rank representation sparse matrix which is used for clustering by fusing or constraining the low-rank representation of different views. Among the low-rank representation-based methods, RMSC [26] adopted the low-rank sparse decomposition for the feature fusion of multi-view data. ILRSO [27] obtained the common sparse representation of different views by learning shared representations in multi-view spectral clustering. MLRR [28] constructed a multi-view subspace clustering incidence matrix which is used to learn the intrinsic feature of the multi-view data by using the angle information of the main direction of the low-rank representation under symmetric conditions. Although the above-mentioned methods incorporated low-rank representation with multi-view learning, they ignore that noise may exist in different views and the clustering results maybe influenced by noisy view. In this paper, a robust clustering method via multi-view low-rank clustering (RCMLRR) is proposed to mitigate the negative influence of the noisy view by weighting. The main contribution of this paper can be summarized in threefold.

1. A robust weighted multi-view low-rank clustering method with consistency structure is proposed based on the low-rank representation-based multi-view subspace clustering.
2. The consistency information of different views is learned by using the similarity feature between views to construct the common structural matrix. The constructed matrix is used to learn the self-representation matrix of each view.
3. The influence of the noisy view is mitigated by setting the difference between the self-representation matrix of each view and the common structured matrix as view weight.

2. Problem formulation

Low-Rank Representation is extended into multi-view scenario in order to utilize multiple information of the object to avoid the influence of noise. However, the noisy view of multi-view data may lead to unideal learning results. The Robust Clustering via Multi-view Low-rank Representation (RCMLRR) method is proposed in this paper, which combines low-rank clustering with multi-view learning and is robust to noises by using consistency structural information of multiple views and adding weights to views.

In this paper, lowercase letters such as x represent vectors and uppercase letters such as X represent matrices. A dataset with n samples of d dimensionality is represented as $X = [x_1, x_2, ..., x_n] \in R^{d\times n}$. Subspace clustering assumes that each sample can be represented by the linear combination of samples in the same subspace, the learned subspace which will be further used for clustering is solved by:

$$\min_{Z,E} \mathcal{L}(X, XZ) + \lambda\Omega(Z),\ s.t.\ X = XZ + E \tag{1}$$

where $Z = [z_1, z_2, ..., z_n] \in R^{n\times n}$ is the reconstruction coefficient matrix, z_i is the subspace representation of x_i, $E \in R^{d\times n}$ is the reconstruction error, $\mathcal{L}(\cdot,\cdot)$ is the loss function that used to measure reconstruction error, $\Omega(\cdot)$ is the regularizer, λ is the parameter that used to balance the loss function and the regularizer. The spectral clustering is applied in $(|Z|+|Z^T|)/2$, $|\cdot|$ is the absolute operator.

The objective function 1 can be extended into multi-view scenario as:

$$\begin{aligned} &\min_{Z^{(v)},E^{(v)}} \sum_{v=1}^{n_v} \mathcal{L}(X^{(v)}, X^{(v)}Z^{(v)}) + \lambda_v \Omega(Z^{(v)}), \\ &\text{s.t.} \quad X^{(v)} = X^{(v)}Z^{(v)} + E^{(v)}, v = 1, 2, \cdots, n_v, \end{aligned} \tag{2}$$

where n_v is the number of views, $X^{(v)}$ is the v-th view $Z^{(v)}$ represents the subspace representation matrix of the v-th view, $E^{(v)}$ is the reconstruction error of the v-th view, λ_v is the balance parameter of the v-th view.

In the proposed SMvLRC method, the low-rank representation of each view in multi-view data is extracted by:

$$\min_{\{Z_i,E_i\}_{i=1}^n} \sum_{i=1}^{n} g(X_i, E_i) + \lambda \sum_{i=1}^{n} f(X_i, Z_i), \tag{3}$$

where $f(\cdot)$ is the learned constraint of the low-rank structure that used the original views, $g(\cdot)$ is the learned constraint of noise, parameter $\lambda > 0$ is used to balance the influence between low-rank representation and regularizer.

Aim to constrain the weight of $x_i^{(v)}$ to $x_j^{(v)}$ is equal to the weight of $x_j^{(v)}$ to $x_i^{(v)}$, the strategy in [28] is used by applying symmetric constraint on multi-view low-rank representation:

$$\begin{aligned} &\min_{\{Z^{(v)},E^{(v)}\}_{v=1}^{n_v}} \sum_{v=1}^{n_v} \left\|E^{(v)}\right\|_{2,1} + \lambda_1 \left\|Z^{(v)}\right\|_*, \\ &\qquad \text{s.t. } X^{(v)} = X^{(v)}Z^{(v)} + E^{(v)}, Z^{(v)} = \left(Z^{(v)}\right)^{\mathrm{T}}, \\ &\qquad\quad v = 1, 2, \cdots, n_v. \end{aligned} \tag{4}$$

Inspired by the method proposed in [29] [30], a structural matrix M which reflects the structure relationship of the samples is used to help Z better reveal the structural information of the data. Meanwhile, in order to ensure that the target structure of each view is consistent, the same structural matrix M is used to constrain each view, function 4 can be improved as:

$$\begin{aligned} &\min_{\{Z^{(v)},E^{(v)}\}_{v=1}^{n_v}} \sum_{v=1}^{n_v} \left\|E^{(v)}\right\|_{2,1} + \lambda_1 \left\|Z^{(v)}\right\|_* + \lambda_2 \|M - Z^{(v)}\|_F^2 \\ &\qquad \text{s.t. } X^{(v)} = X^{(v)}Z^{(v)} + E^{(v)}, Z^{(v)} = \left(Z^{(v)}\right)^{\mathrm{T}}, \\ &\qquad\quad v = 1, 2, \cdots, n_v. \end{aligned} \tag{5}$$

Considering that the feature similarity of corresponding samples is relatively large and multiple views have the same structural matrix, the structural matrix M is formed as:

$$M_{ij} = \frac{\exp(\max\{x_i^{(v)*T} x_j^{(v)*}, v = 1, 2, \cdots, n_v\} - \delta)}{1 + \exp(\max\{x_i^{(v)*T} x_j^{(v)*}, v = 1, 2, \cdots, n_v\} - \delta)}, \tag{6}$$

where $x_i^{(v)*}$ and $x_j^{(v)*}$ are the normalized result of the $v-th$ view $x_i^{(v)}$ and $x_j^{(v)}$, δ is the average value of all $x_i^{(v)*T} {x_j}^{(v)*}$.

Finally, the weight of each view is calculated to reflect the different contributions of different views and to reduce the negative influence of noisy views. In the proposed RCMLRR method, the weight is calculated according to the difference between each view and the structural matrix M by $w_v = \frac{1}{\|Z^{(v)}-M\|_2^2+\epsilon}$, where ϵ is the extreme decimal used to control the value of the denominator is not zero. The normalized weight is $w^{(v)} = \frac{w_v}{\sum_{v=1}^{n_v} w_v}$, and the objective function of RCMLRR is:

$$\begin{aligned} &\min_{\{Z^{(v)},E^{(v)}\}_{v=1}^{n_v}} \sum_{v=1}^{n_v} w^{(v)} \left\|E^{(v)}\right\|_{2,1} + \lambda_1 \left\|Z^{(v)}\right\|_* + \lambda_2 \|M - Z^{(v)}\|_F^2 \\ &\qquad \text{s.t. } X^{(v)} = X^{(v)}Z^{(v)} + E^{(v)}, Z^{(v)} = \left(Z^{(v)}\right)^{\mathrm{T}}, \\ &\qquad\quad v = 1, 2, \cdots, n_v. \end{aligned} \tag{7}$$

3. Optimization

Equation 7 is iteratively solved by the Augmented Lagrange Multipliers (ALM) algorithm [31]. By introducing a relaxation variable $J^{(v)}$, the objective function can be converted to:

$$\begin{aligned} &\min_{Z^{(v)},J^{(v)},E^{(v)}} w^{(v)} \left\|E^{(v)}\right\|_{2,1} + \lambda_1 \left\|J^{(v)}\right\|_* + \lambda_2 \|M - Z^{(v)}\|_F^2 \\ &\text{s.t. } X^{(v)} = X^{(v)}Z^{(v)} + E^{(v)},\ Z^{(v)} = J^{(v)},\ J^{(v)} = \left(J^{(v)}\right)^{\mathrm{T}}. \end{aligned} \tag{8}$$

The augmentation Lagrange function of equation 8 is:

$$\min_{Z^{(v)},J^{(v)},E^{(v)},J^{(v)}=\left(J^{(v)}\right)^T,Y_1,Y_2} w^{(v)}\left\|E^{(v)}\right\|_{2,1}+\lambda_1\left\|J^{(v)}\right\|_*$$
$$+\lambda_2\|M-Z^{(v)}\|_F^2+\operatorname{tr}\left[Y_1^{\mathrm{T}}\left(X^{(v)}-X^{(v)}Z^{(v)}-E^{(v)}\right)\right]$$
$$+\operatorname{tr}\left[Y_2^{\mathrm{T}}\left(Z^{(v)}-J^{(v)}\right)\right]+\frac{\eta}{2}\left(\left\|X^{(v)}-X^{(v)}Z^{(v)}-E^{(v)}\right\|_F^2\right.$$
$$\left.+\left\|Z^{(v)}-J^{(v)}\right\|_F^2\right) \tag{9}$$

where Y_1 and Y_2 are the Lagrange multipliers, $\eta > 0$ is the adaptive penalty parameter. $J^{(v)}, Z^{(v)}, E^{(v)}$ can be solved iteratively. In the optimization process, When optimizing a variable in the $(t+1)$-th iteration, the other two variables in the t-th iteration are fixed.

The Lagrange multipliers and penalty parameters are updated by:

$$\begin{aligned} Y_{1,t+1}^{(v)} &= Y_{1,t}^{(v)}+\eta_t(X^{(v)}-X^{(v)}Z_{t+1}^{(v)}-E_{t+1}),\\ Y_{2,t+1}^{(v)} &= Y_{2,t}^{(v)}+\eta_t(Z_{t+1}^{(v)}-J_{t+1}^{(v)}),\\ \eta_{t+1} &= \min(\rho\eta_t,\eta_{max}), \end{aligned} \tag{10}$$

ρ and η_{max} are constants.

The view weight W is updated by:

$$w^{(v)}=\frac{w_v}{\sum_{v=1}^{n_v} w_v}, \tag{11}$$

where $w_v=\frac{1}{\|Z^{(v)}-M\|_2^2+\epsilon}$.

Finally, the optimization process of solving $\{Z^{(v)}\}$ from equation 7 is show in Algorithm 1.

After the self-representation matrix $Z^{(v)}$ of each view is acquired, the spectral clustering algorithm is used to obtain the final clustering results. Firstly, in order to fuse the self-representation structure of each view, the fused multi-view self-representation matrix is calculated by:

$$Z^*=\sum_{v=1}^{n_v} Z^{(v)} \tag{12}$$

Then, the SVD decomposition $Z^*=U^*S^*V^*$ is imposed on Z^*: and $D=U^*(S^*)^{1/2}$. The affinity matrix A is calculated by:

$$A_{ij}=\left(\frac{d_i^T d_j}{\|d_i\|_2\|d_j\|_2}\right)^{2\alpha} \tag{13}$$

where d_i is the i-th volume of matrix D, α is the control hyperparameter that can ensure each value of the affinity matrix A for subspace clustering is positive.

To obtain the spectral clustering results of the affinity matrix, the corresponding Laplacian matrix is calculated by $L=T^{-\frac{1}{2}}(T-A)T^{-\frac{1}{2}}$, where $T=\sum_{v=1}^{n_v} T^{(v)}$, $T^{(v)}$ is the diagonal matrix with diagonal element: $T_{ii}^{(v)}=\sum_j Z_{ij}^{(v)}$.

Algorithm 1 The ALM algorithm of equation 7

Input Multi-view data: $X=\{X^{(v)}\}_{v=1}^{n_v}$, parameter: λ_1, λ_2, ϵ.

Initialization: $\{J^{(v)}=Z^{(v)}=0\}_{v=1}^{n_v}$, $\{Y_1^{(v)}=Y_2^{(v)}=0\}_{v=1}^{n_v}$, $\rho=1.1$, $\eta_1=\eta_2=1e-2$, $\eta_{max}=10^{10}$, $\epsilon_{stop}=1e-6$.
while The convergence condition is not satisfied: **do**
  for $v=1:n_v$ **do**
    Fix other parameters, update $J^{(v)}$.
    Fix other parameters, update $Z^{(v)}$.
    Fix other parameters, update $E^{(v)}$.
    Update multipliers:

$$\begin{aligned} Y_{1,t+1}^{(v)} &= Y_{1,t}^{(v)}+\eta_t(X^{(v)}-X^{(v)}Z_{t+1}^{(v)}-E_{t+1})\\ Y_{2,t+1}^{(v)} &= Y_{2,t}^{(v)}+\eta_t(Z_{t+1}^{(v)}-J_{t+1}^{(v)}) \end{aligned}$$

  end for
  Update view weight: $w^{(v)}=\frac{w_v}{\sum_{v=1}^{n_v} w_v}$, and:

$$w_v=\frac{1}{\|Z^{(v)}-M\|_2^2+\epsilon}$$

  Update parameter η_{t+1}:

$$\eta_{t+1}=\min(\rho\eta_t,\eta_{max})$$

  Check if the convergence condition is satisfied:
  for $v=1:n_v$ **do**
    if $\|Z^{(v)}-J^{(v)}\|_{max}<\epsilon_{stop}$ and $\|X^{(v)}-X^{(v)}Z^{(v)}-E^{(v)}\|_{max}<\epsilon_{stop}$ **then**
      end
    end if
  end for
end while

Output $\{Z^{(v)}\}_{v=1}^{n_v}$

Then, the eigenvectors $\{v_i\}_{i=1}^k$ corresponding to the k-th minimum eigenvalues of the Laplacian matrix L are obtained by SVD decomposition and the eigenvector is formed as: $V=\{v_i\}_{i=1}^k \in R^{n\times k}$. Finally, n samples with dimensionality k in the eigenvector V are subjected to k-means clustering to obtain the final result.

4. Experiments

4.1 Datasets and experimental setting

The database used to evaluate the performance of the proposed method is introduced as follows.

Yale dataset contains 165 grayscale images of 15 individuals. Each person has 11 images, and the size of each image is 32 × 32 pixels. In the experiments, pixel features, Local Binary Pattern (LBP) [1] and Gabor features [2] are extracted as three views.

3-sources dataset consists of 948 news articles, which are manually classified into 6 categories, covering 416 different news stories. There are 169 stories that appear in these three online news sources: BBC, Reuters, and The Guardian, and each of them can be regarded as a single view of a story. Each story is labeled with a theme from the dataset as its category label.

Four indicators are adopted to evaluate the clustering performance of the proposed algorithm: Clustering Accuracy (ACC); Normalized Mutual Information (NMI), F-measure, Adjusted Rand index (ARI). Among them, the clustering accuracy is defined as $ACC = \frac{1}{n}\sum_{i=1}^{n}\delta(c_i, map(x_i))$, where c_i represents the class label of x_i, $\delta(x, y)$ represents the equivalence between x and y, and $map(x_i)$ is a permutation mapping function that maps each clustering label x_i to one of the class labels among all the class labels.

4.2 Comparison results on the Yale dataset with noises

The proposed RCMLRR in this paper focuses on the clustering problem of data with noisy views. Therefore, in order to test the robustness of the proposed method, the Yale face dataset is selected as the basic dataset in this experiment. Then, a noisy view is constructed by adding different types of occlusions to some of the images in View 1. The specific situations are as follows, 20% of images were randomly selected and augmented with $\frac{1}{2} \times \frac{1}{2}$ size block occlusions in three types: black, white, and random pixels.

TABLE 1. Clustering results of Yale dataset with black-block noise

method	ACC	NMI	F-measure	ARI
LRR_{V1} [16]	56.97	60.33	39.62	35.59
LRR_{V2} [16]	69.70	70.64	53.73	50.60
LRR_{V3} [16]	68.48	69.34	52.38	49.15
LRR_{BestSV} [16]	69.70	70.64	53.73	<u>50.60</u>
LRR_{Concat} [16]	67.88	68.08	50.67	47.33
GMC [4]	61.21	66.21	47.63	43.90
RMSL [5]	<u>73.33</u>	69.06	43.45	38.95
MCLES [6]	69.09	<u>71.86</u>	52.64	49.10
MLRR [28]	67.23	66.12	<u>73.67</u>	38.38
LRSSC [3]	58.63	56.58	60.50	38.41
LMVSC [7]	52.90	51.28	56.33	27.24
RCMLRR (ours)	**75.12**	**73.42**	**80.02**	**55.20**

In the comparison experiment, the proposed method is compared with low-rank representation-based multi-view clustering methods: LRR_{BestSV} [16], LRR_{Concat} [16], GMC [4], RMSL [5], MCLES [6], MLRR [28], LRSSC [3], LMVSC [7]. The parameters of the comparative methods were manually set according to the corresponding papers, and the optimal results of each method were presented. In the proposed RCMLRR, there are a total of three parameters: λ_1, λ_2 and α. Among them, λ_1 is related to the data quality of data from different views, and λ_2 depends on the difference between different views and the common constraints.

The last step of the comparison methods is to run with the k-means algorithm. In this experiment, we set $\lambda_1 = 100$, $\lambda_2 = 0.1$, $\alpha = 1$, the standard k-means implementation provided by MATLAB was adopted, and all the methods shared the same k-means parameters.

TABLE 2. Clustering results of Yale dataset with white-block noise

method	ACC	NMI	F-measure	ARI
LRR_{V1} [16]	59.39	63.16	41.78	37.61
LRR_{V2} [16]	69.70	70.64	53.73	50.60
LRR_{V3} [16]	68.48	69.34	52.38	49.15
LRR_{BestSV} [16]	69.70	70.64	53.73	50.60
LRR_{Concat} [16]	65.45	69.25	51.99	48.72
GMC [4]	63.64	66.45	42.65	38.11
RMSL [5]	<u>75.15</u>	<u>74.88</u>	55.22	51.94
MCLES [6]	70.30	71.44	52.91	49.37
MLRR [28]	71.06	58.04	<u>66.70</u>	<u>54.61</u>
LRSSC [3]	58.63	56.58	60.50	38.41
LMVSC [7]	52.90	51.28	56.33	27.24
RCMLRR (ours)	**78.79**	**76.80**	**82.80**	**58.49**

TABLE 3. Clustering results of Yale dataset with random-block noise

method	ACC	NMI	F-measure	ARI
LRR_{V1} [16]	65.45	65.95	47.82	44.33
LRR_{V2} [16]	69.70	70.64	53.73	50.60
LRR_{V3} [16]	68.48	69.34	52.38	49.15
LRR_{BestSV} [16]	69.70	70.64	53.73	50.60
LRR_{Concat} [16]	67.88	68.08	50.67	47.33
GMC [4]	60.61	66.28	47.11	43.31
RMSL [5]	<u>75.15</u>	74.83	55.69	52.51
MCLES [6]	73.33	<u>76.46</u>	60.18	<u>57.33</u>
MLRR [28]	71.06	58.04	<u>66.70</u>	54.61
LRSSC [3]	58.63	56.58	60.50	38.41
LMVSC [7]	52.90	51.28	56.33	27.24
RCMLRR (ours)	**77.57**	**76.79**	**82.26**	**59.05**

Tables 1, 2, and 3, respectively, show the results obtained by adding $\frac{1}{2} \times \frac{1}{2}$ black block, white block, and random block occlusions to 20% of the images in View 1, and then performing multiview clustering using the algorithm. The results show that adding different occlusions will reduce the performance of the single-view low-rank representation, especially when there is black block occlusion. However, for random block occlusion, the performance degradation of the single-view low-rank representation is not obvious, which proves that the low-rank representation itself can effectively eliminate the influence of Gaussian noise, consistent with the conclusion in the literature [16]. For the data with noisy views, the proposed method has achieved the best results. It is worth noting that the clustering results obtained by the proposed algorithm are similar to those of the data without noise, demonstrating the robustness of the algorithm to noisy views.

4.3 Comparison results on the data without noises

In this section, we employ meticulously designed experimental procedures to comprehensively validate the properties of proposed algorithm. To verify the effectiveness of the proposed method on data without noises, experiments were conducted on the Yale dataset and the 3-sources dataset respectively. Tables 4 and 5 present the experimental comparison results of the Yale dataset and the 3-sources dataset, respectively. In the experiment, the specific values of each parameter in the Yale dataset are $\lambda_1 = 100$, $\lambda_2 = 0.1$, $\alpha = 1$, the specific values of each parameter in the Yale dataset are $\lambda_1 = 2$, $\lambda_2 = 0.1$, $\alpha = 5$.

TABLE 4. Clustering results of different methods on Yale

Method	ACC	NMI	F-measure	ARI
LRR_{BestSV} [16]	70.30	70.93	54.80	51.78
LRR_{Concat} [16]	69.70	70.31	52.77	49.57
GMC [4]	65.45	68.92	44.10	48.01
RMSL [5]	<u>77.58</u>	<u>75.53</u>	48.67	44.50
MCLES [6]	71.52	73.28	58.51	<u>55.66</u>
MLRR [28]	74.55	68.62	77.12	43.18
LRSSC [3]	67.24	72.18	53.10	49.73
LMVSC [7]	76.79	60.96	<u>77.55</u>	51.01
RCMLRR(ours)	**78.18**	**77.69**	**82.58**	**60.78**

TABLE 5. Clustering results of different methods on 3-sources

Method	ACC	NMI	F-measure	ARI
LRR_{BestSV} [16]	60.95	51.96	66.12	44.48
LRR_{Concat} [16]	66.27	63.11	71.20	53.46
GMC [4]	85.21	73.97	83.27	73.19
RMSL [5]	78.46	73.14	80.47	70.38
MCLES [6]	83.02	74.60	84.21	78.28
MLRR [28]	<u>89.29</u>	<u>80.36</u>	<u>89.67</u>	<u>85.22</u>
LRSSC [3]	78.70	69.70	79.30	63.65
LMVSC [7]	76.92	65.38	79.24	60.57
RCMLRR(ours)	**89.35**	**80.49**	**89.76**	**85.23**

As can be seen from the experimental results, compared with the optimal result LRR_{BestSV} of the single-view low-rank representation algorithm, the proposed method has been improved, indicating that the proposed algorithm can effectively address the deficiencies of the single-view low-rank representation. In addition, the proposed method can achieve optimal results on different datasets when compared with other multi-view clustering algorithms according to the showed results, which verifies the effectiveness of the proposed method under noise-free conditions. Specifically, the algorithm demonstrates robust resilience to noise interference. In addition, its efficacy is equally pronounced when applied to datasets without noise, highlighting its versatility and reliability across diverse data conditions.

5. Conclusions

In this paper, a multi-view low-rank representation clustering algorithm based on structural consistency is proposed. Using similarity features among samples, we construct a common structured matrix. This matrix serves as a supervision mechanism during the self-representation matrix learning process of each view, facilitating the extraction of consistent representations across diverse views. To mitigate the influence of views with inferior structures, the weight of each view is calculated. This is achieved by calculating the discrepancies between the self-representation matrix of each view and the common structured matrix. Multi-view clustering experiments conducted under various noise conditions validate the effectiveness of the proposed algorithm. The algorithm significantly reduces the impact of noisy views, enabling the extraction of more robust feature representations.

Acknowledgements

This research was funded by the National Key R&D Program under Grant No. 2022YFC3301000, the Natural Science Foundation of Shandong Province under Grant No. ZR2024QF257, the Science and Technology Support Plan for Youth Innovation of Colleges and Universities of Shandong Province of China under Grant NO. 2023KJ370, and the National Natural Science Foundation of China under Grant No. 62101179.

References

[1] T. Ojala, M. Pietikainen, and T. Maenpaa, “Multiresolution grayscale and rotation invariant texture classification with local binary patterns”, IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 24, no. 7, pp. 971-987, 2002.

[2] M. Lades, Jan. C. Vorbruggen, J. Buhmann, J. Lange, Christoph. V. D. Malsburg, Rolf. P. Wurtz, and W. Konen, “Distortion invariant object recognition in the dynamic link architecture”, IEEE Transactions on Computers, vol. 42, no. 3, pp. 300-311, 1993.

[3] M. Brbić, and I. Kopriva, “Multi-view low-rank sparse subspace clustering”, Pattern Recognition, vol. 73, pp. 247-258, 2018.

[4] Hao Wang, Yan Yang, and Bing Liu, “Gmc: Graph-based multi-view clustering”, IEEE Transactions on Knowledge and Data Engineering, vol. 32, no. 6, pp. 1116-1129, 2019.

[5] Ruihuang Li, Changqing Zhang, Huazhu Fu, Xi Peng, Tianyi Zhou, and Qinghua Hu, “Reciprocal multi-layer subspace learning for multi-view clustering”, Proceeding of ICCV2019 Conference, Seoul, pp. 8172-8180, October 2019.

[6] Man-Sheng Chen, Ling Huang, Chang-Dong Wang, and Dong Huang, “Multi-view clustering in latent embedding space”, Proceeding of AAAI2020 Conference, New York, pp. 3513-3520, February 2020.

[7] Zhao Kang, Wangtao Zhou, Zhitong Zhao, Junming Shao, Meng Han, and Zenglin Xu, “Largescale multi-view subspace clustering in linear time”, Proceeding of AAAI2020 Conference, New York, pp. 4412-4419, February 2020.

[8] P. Cabena, P. Hadjinian, R. Stadler, J. Verhees, and A. Zanasi, Discovering data mining: from concept to implementation, Prentice-Hall, Inc., 1998.

[9] Douglas. C. Montgomery, Elizabeth. A. Peck, and Geoffrey. G. Vining, Introduction to linear regression analysis. John Wiley & Sons, 2021.

[10] F. Hemmatian, and Mohammad. K. Sohrabi, “A survey on classification techniques for opinion mining and sentiment analysis”, Artificial Intelligence Review, vol. 52, no. 3, pp. 1495-1545, 2019.

[11] Dongkuan Xu and Yingjie Tian, “A comprehensive survey of clustering algorithms”, Annals of Data Science, vol. 2, pp. 165-193, 2015.

[12] Y. Bengio, A. Courville, and P. Vincent, “Representation learning: A review and new perspectives”, IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 35, no. 8, pp. 1798-1828, 2013.

[13] G. Hinton, Li Deng, Dong Yu, George. E. Dahl, A. Mohamed, N. Jaitly, A. Senior, V. Vanhoucke, P. Nguyen, Tara. N. Sainath et al., “Deep neural networks for acoustic modeling in speech recognition: The shared views of four research groups”, IEEE Signal Processing Magazine, vol. 29, no. 6, pp. 82-97, 2012.

[14] A. Krizhevsky, I. Sutskever, and Geoffrey. E. Hinton, “Imagenet classification with deep convolutional neural networks”, Communications of the ACM, vol. 60, no. 6, pp. 84-90, 2017.

[15] E. Elhamifar and R. Vidal, “Sparse subspace clustering: Algorithm, theory, and applications”, IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 35, no. 11, pp. 2765-2781, 2013.

[16] Guangcan Liu, Zhouchen Lin, Shuicheng Yan, Ju Sun, Yong Yu, and Yi Ma, “Robust recovery of subspace structures by low-rank representation”, IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 35, no. 1, pp. 171-184, 2012.

[17] Yan Yang and Hao Wang, “Multi-view clustering: A survey”, Big Data Mining and Analytics, vol. 1, no. 2, pp. 83-107, 2018.

[18] K. Chaudhuri, Sham. M. Kakade, K. Livescu, and K. Sridharan, “Multi-view clustering via canonical correlation analysis”, Proceeding of ICML2009 Conference, Montreal, pp. 129-136, June 2009.

[19] Yuhong Guo, “Convex subspace representation learning from multi-view data”, Proceeding of AAAI2013 Conference, Washington, pp. 387-393, July 2013.

[20] Qiang Wang, Yong Dou, Xinwang Liu, Qi Lv, and Shijie Li, “Multi-view clustering with extreme learning machine”, Neurocomputing, vol. 214, pp. 483-494, 2016.

[21] Hongchang Gao, Feiping Nie, Xuelong Li, and Heng Huang, “Multi-view subspace clustering”, Proceeding of ICCV2015 Conference, Santiago, pp. 4238-4246, December 2015.

[22] Xin Zhang, D. Phung, S. Venkatesh, D.-S. Pham, and Wanquan Liu, “Multi-view subspace clustering for face images”, Proceeding of DICTA2015 Conference, Adelaide, pp. 1-7, November 2015.

[23] Changqing Zhang, Huazhu Fu, Si Liu, Guangcan Liu, and Xiaochun Cao, “Low-rank tensor constrained multiview subspace clustering”, Proceeding of ICCV2015 Conference, Santiago, pp. 1582-1590, December 2015.

[24] Changqing Zhang, Huazhu Fu, Jing Wang, Wen Li, Xiaochun Cao, and Qinghua Hu, “Tensorized multi-view subspace representation learning”, International Journal of Computer Vision, vol. 128, no. 8-9, pp. 2344-2361, 2020.

[25] Yongyong Chen, Xiaolin Xiao, and Yicong Zhou, “Jointly learning kernel representation tensor and affinity matrix for multiview clustering”, IEEE Transactions on Multimedia, vol. 22, no. 8, pp. 1985-1997, 2019.

[26] Rongkai Xia, Yan Pan, Lei Du, and Jian Yin, “Robust multiview spectral clustering via low-rank and sparse decomposition”, Proceeding of AAAI2014 Conference, Québec, pp. 2149-2155, July 2014.

[27] Yang Wang, Wenjie Zhang, Lin Wu, Xuemin Lin, Meng Fang, and Shirui Pan, “Iterative views agreement: An iterative low-rank based structured optimization method to multi-view spectral clustering”, arXiv preprint arXiv:1608.05560, 2016.

[28] Jie Chen, Shengxiang Yang, Hua Mao, and C. Fahy, “Multiview subspace clustering using low-rank representation”, IEEE Transactions on Cybernetics, vol. 52, no. 11, pp. 12 364-12 378, 2021.

[29] Kewei Tang, Risheng Liu, Zhixun Su, and Jie Zhang, “Structure-constrained lowrank representation”, IEEE Transactions on Neural Networks and Learning Systems, vol. 25, no. 12, pp. 2167-2179, 2014.

[30] Lai Wei, Yan Zhang, Jun Yin, Rigui Zhou, Changming Zhu, and Xiafeng Zhang, “An improved structured low-rank representation for disjoint subspace segmentation”, Neural Processing Letters, vol. 50, pp. 1035-1050, 2019.

[31] Zhouchen Lin, Risheng Liu, and Zhixun Su, “Linearized alternating direction method with adaptive penalty for low-rank representation”, Proceeding of NeurIPS2011 Conference, Granada, pp. 612–620, December 2011.